# REVIEWS

**Viscous Flow Theory, I—Laminar Flow,** by SHIH-I PAI. Princeton: D. Van Nostrand Co., 1956. 384 pp. $7.75 or 58*s*.

In the study of real fluids, this has been the half-century of boundary layer theory. It is remarkable how Prandtl's discovery still dominates research in viscous flow, as it has done since 1904, though one may suspect that the possible extensions of boundary layer theory are at last nearly exhausted, and that the really exciting discoveries await us in problems in which the boundary layer, if present at all, is but one component of the flow field. Oseen swam against this current for years, not finding it necessary in his *Hydrodynamik* of 1927 even to mention Prandtl or his Grenzschicht. But with a few other exceptions the history of viscous flow theory since the turn of the century has been in large part the record of the development of boundary layer theory, following paths that to a remarkable extent were at least sketched in outline by Prandtl and his students in the first few pioneering years.

It is perhaps not surprising, then, that the book under review, which is the first of a two-part work on viscous flow theory (the second being devoted to turbulent flow), is concerned mainly with the theory of the laminar boundary layer. Indeed, aside from two short and rather disappointing chapters on other topics, it is devoted entirely to a thoroughly modern survey of laminar boundary layer theory, together with the requisite introductory chapters. It is not intended to be a comprehensive treatise in the style of Schlichting's *Grenzschicht-Theorie*, but a textbook for advanced students of engineering and aerodynamics. The account is purely theoretical, with no enlightenment ever sought from experiment.

The great virtue of the book is that it is the first to assign a central role to compressibility and heat conduction. In this respect it is strongest where Schlichting's treatise is weakest; for, even in the expanded English translation, Schlichting considers an incompressible fluid from the outset, and defers to the advent of high-speed flight only in two appended chapters. (Goldstein's *Modern Developments in Fluid Dynamics*, written as it was at the dawn of the age of compressibility, could not reasonably have attempted a unified treatment of compressibility and heat conduction.) To be sure, Dr. Pai is forced more often than not to retrench in view of mathematical complexity, and to consider a particular problem only in the special case of incompressible flow. But his eye is constantly on the compressible fluid. This is undoubtedly the modern approach, to be taken by future writers.

If one thus breaks with tradition and starts from the Navier–Stokes equations for a compressible heat-conducting gas, it is imperative that he understand precisely how compressibility enters the problem, and how the often necessary specialization to incompressible flow is to be made. The supersonic aerodynamicist, who automatically associates compressibility with Mach number, may be surprised to realize that for a heat-conducting fluid there is a second way in which compressibility can appear. No matter how slight the Mach number, heating of a body will induce a temperature field in the flow and consequent variations of density. Unfortunately, such important distinctions are not always clearly made in the present book. Nor are they perhaps entirely clear to the author, who remarks (p. 49): " Another current practice in finding the temperature distribution approximately is to drop the viscous dissipation terms in the energy equation, and then to solve for the temperature distribution. Strictly speaking, such an approximation is not logical, because the terms of heat conduction and vicous dissipation are of the same order of magnitude. But for practical purposes, when both the velocity of the flow and the temperature gradients are small, such an approximation seems to give reasonably good results."

Now the fact of the matter is that conduction and dissipation are not of the same order of magnitude in general, and that there are important classes of problems in which it is an entirely logical approximation to drop the dissipation term (and the more so the larger the temperature gradients). Consider for definiteness a stream of perfect gas flowing at Mach number $M$ past a heated body. Introducing dimensionless variables in the energy equation (or solving an example) shows that both the dissipation and the enthalpy changes due to pressure gradients are proportional to $M^2$. Hence dissipation and also pressure terms in the energy equation can be neglected strictly at $M = 0$ or approximately when $M^2$ is very small. If, in addition, the maximum temperature difference $\Delta T$ is small compared with the temperature $T$ itself, then density variations are slight, and to a first approximation the momentum and energy equations are not coupled, so that the heat is transported by the incompressible velocity field. (This is readily verified by expanding all flow quantities in powers of $\Delta T/T$ and retaining only linear terms.)

If the body is insulated (the thermometer problem), viscous dissipation is the only source of heat, so that a problem exists only for $M > 0$. If $M^2$ is small, the heat is again carried by the incompressible velocity field to a first approximation. The combined problem in which heat is supplied both by transfer from the body and by dissipation can be handled with a double series expansion, which shows that to a first approximation for small $\Delta T/T$ and $M^2$ the solution of the thermometer problem can simply be added to the solution of the heat transfer problem for $M = 0$, and that non-linearities as well as coupling between the momentum and energy equations arise only in the second-order terms.

It should be pointed out that the above conclusion that dissipation is strictly negligible at $M = 0$ holds only for flows with a definite stream. In problems of free convection the dissipation can be neglected only as a first approximation for slight heating, because the velocities are proportional to $\Delta T/T$ and the dissipation is consequently of order $(\Delta T/T)^2$. It must be admitted that these matters are not well explained in most existing books on viscous flow theory, or in many research papers. (They are very carefully discussed in Lagerstrom's forthcoming article on laminar flow in the Princeton series, and briefly but precisely in Lighthill's paper in *Proc. Roy. Soc.* A, vol. 202, 1950, p. 359.)

The first hundred pages of the present book are introductory in nature, covering basic thermodynamics and kinetic theory, the laws of friction and heat conduction, the derivation of the equations of motion, a number of illustrative exact solutions (mainly incompressible), dynamic similitude, and a general discussion of the Navier–Stokes equations, including Burgers' model, linearization, and the limits of low and high Reynolds number. There follows a short chapter on slow motion, which has a curiously old-fashioned tone. Indeed, there are no references to work done since 1910, which is a pity, since the realm of Stokes and Oseen flow is one in which some interesting discoveries have recently been made, with more expected to follow.

The main bulk of the book, some 200 pages, is devoted to a generally admirable survey of laminar boundary layer theory. This is the field of the author's own research, is clearly where his heart lies, and is by far the strongest part of the book. The emphasis is on compressible flow where possible, including such recent matters as the hypersonic boundary layer and interaction with a shock wave. Successive chapters treat the boundary layer equations, exact solutions, approximate methods, axisymmetric and three-dimensional boundary layers, unsteady boundary layers (including a sketch of the stability theory), and boundary layers with suction and injection. Aside from Kuerti's survey in the second volume of *Advances in Applied Mechanics*, there is no comparable summary of compressible laminar boundary layer theory, and I would recommend it with only minor quibbles. I would only protest that more than the necessary two boundary conditions are provided for the energy equation on pages 148–149, and that of these the requirement of equal normal temperature gradients in the gas and body at their juncture is incorrect. (Note also the unfortunate typographic loss, on page 178, of a radical sign in Blasius' famous formula for flat-plate skin friction.) In particular, the author displays considerable skill in summarizing the various complicated extensions to a compressible fluid of the Kármán–Pohlhausen approximation.

He breaks away from the boundary layer approximation again in the last chapter, which is entitled " Linearized theory of viscous compressible fluid ". Although mainly a survey of Lagerstrom, Cole, and Trilling's study of compressible Oseen flow, it also includes, among other incidentals, an overly-condensed account of Kuo's second approximation to Blasius' solution for the boundary layer on a finite flat plate, from which the main result—the numerical coefficient of the second term (it is $a_2 = 4{\cdot}12$)—is unaccountably omitted. Unfortunately, the author has thought it necessary to copy some ten pages of his book literally word for word from the report of Lagerstrom *et al.* (In his only major departure from that text, he thoroughly garbles the characteristic condition for a second-order partial differential equation.) Now it is perhaps a matter of taste whether one chooses to expound another's theory by quoting him verbatim and at length; this seems to be a growing trend in the literature of fluid mechanics, for I have found three well-known writers doing so within the last year. But it is certainly another, and quite indefensible, matter when the present author concocts introductory pages that are ostensibly his own by assembling a pastiche of sentences and paragraphs lifted from the Lagerstrom report.

This is, then, a book with some serious flaws, written in a style that is often derivative, and concerned more with examples than ideas. Despite its title, it concentrates on the boundary layer approximation to the virtual exclusion of other facets of viscous flow theory. Yet, admitting these limitations, it is the best available survey of laminar boundary layer theory from the modern point of view. The student who arms himself also with at least the first of Goldstein's two masterly volumes, to provide the physical insight and appeal to experiment that the present book lacks, should find it a useful guide to that subject.

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**Molecular Flow of Gases,** by G. N. PATTERSON. New York: John Wiley & Sons, 1956. 217 pp. \$7.50 or 60*s*.

The molecular viewpoint in studies of gas flows is by no means a new area of interest. It goes back at least to the studies of Kundt, Warburg and Maxwell (1875–1879) on low-speed slip flow, and the later work of Knudsen and Smoluchowski (1910–1911) on low-speed free molecule flow. However, the increase in the amount of research in molecular gas flows since World War II is most impressive. Over and above a proportionate share of the post-war growth of all branches of science, the increasing importance of the molecular viewpoint in many problems has attracted the attention of more and more scientists from the disciplines of physics, chemistry and astronomy.

There is a wide range of problems requiring a consideration of the molecular structure of moving gases. Such considerations are necessary in the study of colliding galaxies, supernovae, stellar atmospheres, and meteors, in the development of high-speed, long-range missiles, in the development of controlled thermonuclear reactions, and in the study of shock-induced chemical reactions. An illustration of the kind of problem to be dealt with is provided by a simple shock transition. For weak shock waves in a moderately dense monatomic gas, the shock transition zone (the region between the shock front and the attainment of equilibrium in the shocked state) is so thin as to defy detailed experimental study, the gas behaves ideally, and the transition can be dealt with adequately by regarding it as a discontinuity in a continuum fluid flow problem. Conservation laws are here sufficient for a determination of the change in flow properties across the transition. At low densities in a monatomic gas the density profile through the shock thickens in proportion to the increased mean free path in the gas, for a certain number of collisions are required for equilibration of translational energy of the molecules. (It is interesting to note here that a shock profile derived from the Navier–Stokes equation with the continuum viewpoint represents the observed transitions better than the best solutions of the Boltzmann equation yet available.) With stronger shocks in a diatomic gas the transition zone is thickened further, as the gas requires many more collisions to equilibrate rotational and vibrational degrees of freedom than to equilibrate translational kinetic energy. Here we must not only consider the Boltzmann equation with inelastic collisions, but must modify the ideal gas form of the internal energy to include thermodynamic properties of the specific gas under study, in order to specify correctly the final equilibrium state. For still stronger shocks we must include molecular dissociation, ionization and other gas phase reactions. Much work needs to be done before we understand on a theoretical basis even such a basic problem as the shock transition in molecular gas flows.

There appear to be several stages of sophistication involved in considering gas flows on a molecular basis. There is firstly a purely kinetic viewpoint in which one assumes elastic collisions and a model for the intermolecular forces in the gas. If the collisions cause at most a small deviation from a Maxwell velocity distribution, then one can use the Boltzmann equation and the Maxwell equations of transfer to develop the fluid equations of motion, including transport properties. Boundary conditions at a surface in the flow also may be developed on this basis. A more sophisticated development would describe gas flows under the condition that the molecular collisions are inelastic. Here the collisions not only alter materially the velocity distribution function, but involve energy transfer into internal excitation, dissociation, ionization, recombination, and so on. In a still more sophisticated analysis one might attempt (although whether it would be profitable to do so is debatable) to deal with turbulent flows, in which one might combine consideration of microscopic, random molecular motions with the ordered motion in eddies having dimensions of many gas mean free paths. Other more complex flow problems include the motion of highly ionized gases involving collective behaviour due to long range forces, and the motion of very dense gases in which one may not be able to limit the analysis to bimolecular collisions only and in which collisions may not be isentropic because of the finer details of intermolecular forces.

Professor Patterson's book is concerned primarily with the first stage of sophistication mentioned above. The book has as its stated purpose the provision of a transition from the continuum to the molecular viewpoint in fluid mechanics, and provides perhaps as elementary an introduction to the kinetic theory of non-uniform gases as can reasonably be expected for those interested principally in fluid mechanics. It might be read with profit by those who would later undertake study of such works as *Mathematical Theory of Non-Uniform Gases* by Chapman and Cowling or *Molecular Theory of Gases and Liquids* by Hirschfelder, Curtiss and Bird.

The book begins with a brief discussion of basic concepts in kinetic theory, including an introduction to the Boltzmann integro-differential equation and the Maxwell transfer equation. A very clear discussion is given of molecular collisions. Next the author describes the solution of these equations for gases represented by perfectly elastic spherically symmetric particles, having no internal degrees of freedom. Collisions of such particles do not alter the velocity distribution function, the gas is in a steady state, and the Boltzmann equation leads directly to the Maxwellian velocity distribution. The transfer equation then leads to the equations of isentropic flow. Properties of a Maxwellian gas are then discussed. As part of the isentropic flow discussion the author develops at some length the non-stationary expansion wave.

The major part of the book is devoted to slightly non-isentropic flows, i.e. flows in which small gradients in composition, temperature, and density occur, and in which the velocity distribution function deviates slightly from the Maxwellian. To describe such flows one assumes a repulsive force field for a point particle having no internal degrees of freedom. Using a method due to Grad, the author obtains a perturbation solution to first order of the Boltzmann equation for a gas not in a shady state. The resulting transport properties are similar to the Chapman–Enskog result. The theory is then applied to weak shock transitions and boundary layer flows. Comparisons are made with experimental studies of slightly non-isentropic flows. A very brief discussion is given of the flow alterations resulting from real gas effects, i.e. internal excitation of atoms and molecules, dissociation, ionization, etc. Finally there is a description of the present state of the theory of slip flow and of free molecule flows. Appendices include some mathematical relations which occur in kinetic theory problems, a discussion of first-order ordinary and partial differential equations, and a concise statement of the procedure leading to the Chapman–Enskog result and the Burnett approximation.

Several features and flaws of the book invite comment. The author has achieved his goal of giving an introduction to the flow of gases with molecular structure. The discussion of molecular collisions and the development of the flow of rarified gases are presented with particular clarity. The exposition of expansion waves and characteristics is unnecessarily long, not too pertinent to the main thesis of the book, and the material covered is readily available to the reader elsewhere. The discussion of gas flows in which molecules are excited, dissociated or ionized, or in which chemical reactions may occur, is much too brief. What discussion is given is cursory and fails to convey to the reader what can be accomplished by an analysis of high temperature and high velocity flows in terms of thermodynamics and chemical kinetics. For example, one finds the familiar equilibrium constant applied to a dissociation reaction referred to as "a characteristic quantity relevant to dissociation". The appendices could be eliminated without any particular loss to the book, except that the brief mention of the Burnett approximation might profitably be expanded and incorporated into the text as part of a discussion of the general problem of dealing with the Boltzmann equation. The author does not even hint that there is at present considerable interest in the molecular description of the flow of conducting gases in magnetic fields.

The flaws mentioned nevertheless are minor, and do not detract seriously from the general worth of the book as a useful text for introduction of the student of gas dynamics to the molecular viewpoint.

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